$(x+2)^2$

$(x+2)(x+2)$

$x^2 + \underbrace{2x + 2x}$ $+ 4$

$x^2 + 4x + 4$

$-x^2$ when x=-4

$-(-4)^2$

-16

$(2)(3)^2 = 2(3)^2$

$2 \cdot 9$

18

$\sqrt{32}$

$\sqrt{16 \cdot 2}$

$4\sqrt{2}$

Matrices

Matrices

Definition

rectangular array of data enclosed in a set of brackets.

Facts/Characteristics

Dimension = rows x columns

Use capitol letters to Name

Each # in a matrix is called an entry

Examples

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 6 \\ 4 & 7 \\ 6 & 8 \end{bmatrix}$

Non-examples

$4x+2y=6$

$\{1, 2, 3, 4\}$

$|6|$

$$A = \begin{bmatrix} 3 & 5 & 11 \\ -6 & 4 & 5 \\ -31 & -42 & -1 \\ 0 & 8 & 9 \end{bmatrix}$$

4 rows

3 columns

4×3

$$C = \begin{bmatrix} \boxed{4} & -2 & 5 \\ -3 & 6 & 9 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & -2 & 5 \\ -3 & 6 & 9 \end{bmatrix}$$

$e_{1,1} = 4$ $\quad e_{1,2} = -2$

$e_{2,3} = 9$ $\quad e_{2,1} = -3$

- To add or subtract matrices, you must have the same dimensions

$$A = \begin{bmatrix} 3 & 5 & 11 \\ -6 & 4 & 5 \\ -31 & -42 & -1 \\ 0 & 8 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 32 & 24 & 14 \\ 8 & -5 & -14 \\ 12 & -7 & 3 \\ -10 & 11 & 6 \end{bmatrix}$$

- What is A+B=

4 x 3

$$A+B = \begin{bmatrix} 35 & 29 & 25 \\ 2 & -1 & -9 \\ -19 & -49 & 2 \\ -10 & 19 & 15 \end{bmatrix}$$

Scalar Multiplication

- Scalar Multiplication takes the elements of one matrix and multiplies it by some factor (not the same as multiplying one matrix by another)
- ½C=

$$C = \begin{bmatrix} 4 & -2 & 5 \\ -3 & 6 & 9 \end{bmatrix}$$

$$\frac{1}{2}C = \frac{1}{2}\begin{bmatrix} 4 & -2 & 5 \\ -3 & 6 & 9 \end{bmatrix}$$

$$\frac{1}{2}C = \begin{bmatrix} 2 & -1 & \frac{5}{2} \\ -\frac{3}{2} & 3 & \frac{9}{2} \end{bmatrix}$$

Try the following....

$$A=\begin{bmatrix} 2 & -2 & 25 \\ -1 & 5 & 19 \end{bmatrix} \qquad B=\begin{bmatrix} 40 & 32 & 51 \\ -13 & -23 & 12 \end{bmatrix}$$

$$C=\begin{bmatrix} 2 & 3 & 4 \\ -6 & 7 & 4 \\ 10 & 5 & -1 \\ 0 & 3 & -9 \end{bmatrix} \quad D=\begin{bmatrix} 21 & 1 & 5 \\ -3 & 2 & 7 \\ -4 & 3 & 12 \\ -8 & 4 & 3 \end{bmatrix} \quad E=\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

1. A+2B= 2. 3C-4D= 3. 2A+3E= 4. 3D-2C+4E=

$$A+2B=\begin{bmatrix} 2 & -2 & 25 \\ -1 & 5 & 19 \end{bmatrix}+2\begin{bmatrix} 40 & 32 & 51 \\ -13 & -23 & 12 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -2 & 25 \\ -1 & 5 & 19 \end{bmatrix}+\begin{bmatrix} 80 & 64 & 102 \\ -26 & -46 & 24 \end{bmatrix}$$

$$=\begin{bmatrix} 82 & 62 & 127 \\ -27 & -41 & 43 \end{bmatrix}$$

Matrix Multiplication

- Used to multiply one matrix by another matrix—dimensions do not have to be the same (but the inside numbers have to match)
- EX. a 2X3 and a 3X4 can be multiplied together and the result will be a 2X4 (outside numbers)

✓ 3x4 4x5

NO 2x3 4x2

Multiplying Matrices

$$R = \begin{bmatrix} 2 & 3 & 4 \\ -6 & 7 & 4 \\ 10 & 5 & -1 \\ 0 & 3 & -9 \end{bmatrix} \qquad S = \begin{bmatrix} 2 \\ 3 \\ 11 \end{bmatrix}$$

4 x 3 3 x 1

RS=

$$\begin{bmatrix} 2 & 3 & 4 \\ -6 & 7 & 4 \\ 10 & 5 & -1 \\ 0 & 3 & -9 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 11 \end{bmatrix}$$

$\cancel{SR}$= 3x$\boxed{1 \quad 4}$x3

$2 \cdot 2 = 4$
$3 \cdot 3 = 9$
$4 \cdot 11 = 44$
+ 57

$10 \cdot 2 = 20$
$5 \cdot 3 = 15$
$-1 \cdot 11 = -11$
24

$-6 \cdot 2 = -12$
$7 \cdot 3 = 21$
$4 \cdot 11 = 44$
53

$0 \cdot 2 = 0$
$3 \cdot 3 = 9$
$-9 \cdot 11 = -99$
-90

$$\boxed{\begin{bmatrix} 57 \\ 53 \\ 24 \\ -90 \end{bmatrix}}$$

You try....

$$A=\begin{bmatrix}20 & 2 & -2\\ -1 & 3 & 4\end{bmatrix} \quad B=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$$

What is BA?

$$\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}\begin{bmatrix}20 & 2 & -2\\ -1 & 3 & 4\end{bmatrix}$$

$2 \times \boxed{2 \quad 2} \times 3$

$2 \cdot 20 = 40$
$3 \cdot -1 = -3$
37

$2 \cdot 2 = 4$
$3 \cdot 3 = 9$
13

$2 \cdot -2 = -4$
$3 \cdot 4 = 12$
8

8
15

$$\begin{bmatrix}37 & 13 & 8\\ 75 & 23 & 12\end{bmatrix}$$

Is AB possible?

Solving for unknowns in a matrix

- Two matrices are equal if they have the same dimensions and the corresponding entries are equivalent

$$\begin{bmatrix} 2x+4 & 5 \\ -2 & -3y+5 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ -2 & 5y-3 \end{bmatrix}$$

$2x+\cancel{4}=12$
-4
$2x=8$
$\boxed{x=4}$

$-\cancel{3y}+5=5y-3$
$+3y$
$5=8y-\cancel{3}$
$+3$
$8=8y$
$\boxed{y=1}$

Solving systems with matrix equations

- A financial manager wants to put $50,000 in an investment for a client, some in a low risk one earning 5% and some in a high risk one earning 14%. How much money should be invested at each interest rate to earn $5000 in interest per year?

- Set up system: Let x represent the amount in the low earning investment and y represent the amount in the high earning investment

- X+Y=50000
- .05x+.14y=5000

Set up Matrix equation

Coefficient Variable Answer

$$A = \begin{bmatrix} 1 & 1 \\ .05 & .14 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 50000 \\ 5000 \end{bmatrix}$$

- If A represents the coefficients of your variables, X represents the variables, and B represents the constants, then…

$$1x + 1y = 50{,}000$$
$$.05x + .14y = 5{,}000$$

$$\cancel{A^{-1} \cdot} \boxed{AX=B}\; A^{-1}$$

- To solve for x, you would divide by A (which is the same as multiplying by the inverse)--do this on both sides

- So $x=A^{-1}B$

X = The inverse of the coeff. matrix times Answer Matrix

SOLVING SYSTEMS WITH MATRICES

1. 2nd Matrix
2. ▷ ▷ To edit
3. Choose 1: [A]. Enter the dimensions and all entries.
4. Repeat until you have entered all matrices.
5. 2nd Quit (home screen)
6. 2nd Matrix 1: [A]
7. X^{-1}
8. 2nd Matrix 2: [B]
9. enter

Solve this using matrix equations

$$\frac{1}{3}r + \frac{2}{3}s = 5$$

$$\frac{2}{3}r - \frac{1}{2}s = -3$$

$$\underbrace{\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{2} \end{bmatrix}}_{2\times 2} \begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} 5 \\ -3 \end{bmatrix}}_{2\times 1}$$

Solve using matrices

$$x + y + z = 6$$

$$2x + y - 4z = -15$$

$$5x - 3y + z = -10$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -4 \\ 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -15 \\ -10 \end{bmatrix}$$